PMM U.S.S.R.,Vol.46,pp.34-40
0021-8928/83/1 0034 $7.50/0

UDC 532.526

FLOW PAST A CONVEX CORNER WITH A FREE STREAMLINE UNDER LARGE SUBSONIC VELOCITIES*

V.N. DIESPEROV

The flow past a convex corner of a subsonic gas flow with a free stream line converging to the corner point is studied. A solution is constructed in the hodograph plane of a nonlinear equation system for perturbed velocities, describing in the first approximation a potential flow in a neighborhood of the corner point and usually used in transonic gasdynamics /1/. This solution as the Mach number $M_0 \to 0$ at the corner point passes in a continuous manner into the solution of the Laplace equation taken as the limit equation when investigating the separation of a laminar boundary layer of an incompressible fluid /2,3/, while as $M_0 \to 1$, into a selfsimilar solution of the Fal'kovich—Kármán equation with a selfsimilarity parameter $n = 6/5$ /4/, which describes the external potential current under a sonic separation from the corner point. The main attention is paid to the case when the approach stream velocity in it becomes close to sonic and an essential role in the formation of the flow begins to be played by the medium's compressibility. The equation of the free streamline in the whole range of variation of the Mach number, $0 \leqslant M_0 \leqslant 1$, remains one and the same to within a proportionality coefficient. The pressure gradient specified by the exact solution of the nonlinear equations is favourable and tends to infinity as the corner point is approached. Under an interaction of such an external potential flow with the boundary layer there is formed a domain of free interaction /5 — 7/. Estimates are obtained, connecting the Reynolds number with the magnitude of the difference $1 - M_0^2$, under which either the theory of separation of an incompressible fluid /3/ or the theory of sonic separation can be used in the first approximation.

By AO we denote the generator of a corner abutting an incident irrotational flow of an ideal gas whose velocity is assumed subsonic and by OB we denote a free streamline converging to the corner point O (Fig.1). We introduce a Cartesian coordinate system x, y whose negative semiaxis coincides with AO and a Mises coordinate system x, ψ. By q_x and q_y we denote the projections of the velocity vector onto the x- and y-axis, respectively; a is the velocity of sound, ρ is the density, p is the pressure. All equations below are assumed dimensionless. The values of the flow parameters at the corner point are taken as typical and marked by zero subscripts. We are required to find solutions of the system of Euler equations and of the state equations, closing it, of an ideal gas in the domain $\psi > 0$, such that the nonflow condition $q_y = 0$ when $\psi = 0$ and $x > 0$, while the flow velocity equals its own value at the corner point when $\psi = 0$ and $x > 0$, i.e., $(q_x^2 + q_y^2)^{1/2} = 1$.

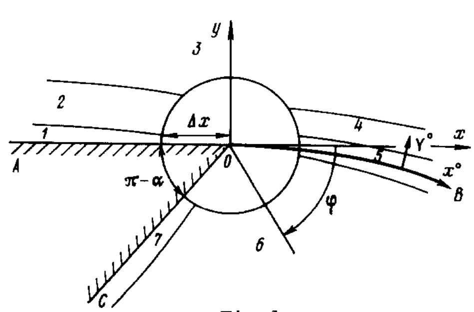

Fig.1

By v_x and v_y we denote the components of the perturbed velocities, $q_x = 1 + v_x$, $q_y = v_y$. In a neighborhood of the corner point O, in which the components of peturbed velocities v_x and v_y are small in comparison with unity, we can construct a system of Euler equations and boundary conditions and represent it in the first approximation as

$$\varepsilon \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial \psi} - (1+\gamma) M_0^2 v_x \frac{\partial v_x}{\partial x} = 0, \quad \varepsilon = 1 - M_0^2 \quad (1)$$

$$\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial \psi} = 0$$

$$v_y \to 0, \ \psi \to 0, \ x < 0; \ v_x \to 0, \ \psi \to 0, \ x > 0 \quad (2)$$

*Prikl.Matem.Mekhan.,46,No.1,41-49,1982

Here γ is the specific heat ratio. It is most convenient to study the solvability of problem (1), (2) in the hodograph plane. The equation system (1) can be inverted and for the determination of the flow function ψ we obtain the linear equation

$$\varepsilon\frac{\partial^2\psi}{\partial v^2}+\frac{\partial^2\psi}{\partial u^2}-M_0^2u\frac{\partial^2\psi}{\partial v^2}=0,\quad u=(1+\gamma)v_x,\quad v=(1+\gamma)v_y \tag{3}$$

We are required to find the solution $\psi>0$ of Eq. (3) in the domain $u<0$, $v<0$, satisfying the conditions

$$\psi=0,\ v=0,\ u\leqslant 0;\ \psi=0,\ u=0,\ v\leqslant 0 \tag{4}$$

After ψ has been determined the quantity x is found by integrating the relations

$$\frac{\partial x}{\partial v}=\frac{\partial\psi}{\partial u},\quad \frac{\partial x}{\partial u}=-(\varepsilon-M_0^2u)\frac{\partial\psi}{\partial v} \tag{5}$$

When $M_0=1$ and $\varepsilon=0$, i.e., when the velocity of sound has been reached at the corner point, Eq. (3) turns into the Tricomi equation. The boundary-value problem (3), (4) for it was studied in /4/, by analogy with which we shall seek the solution of problem (3), (4) when $M_0\neq 1$ as a sum of eigenfunctions

$$\psi=\sum_{l=0}\psi_l=\sum_{i,k}d_{ik}u^iv^k\qquad (k>0,\ i>0)$$

The result of substituting it into Eq. (3) is

$$\psi=d_{11}uv+d_{31}\left[-\varepsilon vu^3+v^3u+\frac{1}{2}M_0^2vu^4\right]+\ldots \tag{6}$$

The constant d_{11} cannot equal zero since the eigenfunctions ψ_l $(l>0)$ change sign in the domain $u<0$, $v<0$. Substituting (6) into (5) and integrating, we obtain a solution in the first approximation of problem (1), (2) with $M_0\neq 1$

$$x=\frac{d_{11}}{2}\left[v^2-\varepsilon u^2+\frac{2}{3}M_0^2u^3\right],\qquad \psi=d_{11}uv \tag{7}$$

When $M_0=0$ the solution becomes the solution of the Laplace equation, while when $M_0=1$, of the Tricomi equation /4/. As should be expected, as $x\to 0$ and for any $M_0\neq 1$ the nature of the singularity is determined by the terms in solution (7), corresponding to the solution of the Laplace equation

$$\varepsilon\frac{\partial^2\psi}{\partial v^2}+\frac{\partial^2\psi}{\partial u^2}=0 \tag{8}$$

In the first approximation it describes the flow around problem in the linear approximation. To detect the influence of the nonlinear term as a function of number M_0 on the character of the flow, we consider the solution of problem (8), (2). It is given by formulas

$$\psi=d_{11}uv+\sum_{k\geqslant 2}\operatorname{Re}\left[(D_\omega^{(1)}+iD_\omega^{(2)})(\sqrt{\varepsilon}u+iv)^\omega\right] \tag{9}$$

$$u=-\sqrt{\frac{2}{d_{11}\varepsilon}}(-x)^{1/2}+O\left[(-x)^\beta\right]+\ldots,$$

$$\psi\to 0,\quad x<0,\quad \beta=\frac{\omega-1}{2}$$

An expansion of velocity u in a neighborhood of the negative axis was obtained in /2/. The eigenfunctions in the expansion of ψ are functions with even powers $\omega=2k$ $(k\geqslant 2)$. If however, we allow for the nonlinearity of the boundary condition on the free streamline, then in the expansion of u when $\psi=0, x<0$ it is necessary to introduce terms with integer powers of x, which corresponds to the odd values $\omega=2k+1$ $(k\geqslant 1)$. Formulas (7) and (9) show that for any subsonic velocities and for the velocity of sound in the corner point the form of the free streamline is specified by the equation

$$y=-\frac{2}{3(1+\gamma)}\sqrt{\frac{2}{d_{11}}}x^{3/2}+\ldots,\quad x>0$$

and coincides with the form of the free streamline in the case of the separation of an incompressible liquid /2/. At the same time the nature of the behavior of the longitudinal velocity component u in the first approximation in a neighborhood of the negative x-axis, under the linear description (8), (9) and with $M_0 \neq 1$ (which is analogous to the nature of the behavior in the case of an incompressible liquid $\varepsilon = 1$, $M_0 = 0$), is essentially different from the nonlinear (7). This difference is particularly noticeable as $M_0 \to 1$. The equations for the perturbed velocities are valid if $(-u) \sim 1$. When $(-u) \sim 1$ the linear theory yields $(-x) \sim \varepsilon$, whereas solution (7) of the nonlinear equation system (1) yields $(-x) \sim 1 + \varepsilon$. The pressure gradients for these values of u are, respectively, $dp / dx \sim -\varepsilon^{-1}$ and $dp / dx \sim -1$.

Let us investigate in detail the behavior of the velocity components u and of the pressure gradient on wall AO. From formula (7) when $\psi = 0, v = 0$ we obtain

$$(-u)^3 + \delta(-u)^2 - X = 0, \quad \delta = \frac{3}{2M_0^2}\varepsilon, \quad X = -\frac{3}{d_{11}M_0^2}x \tag{10}$$

The solution of the cubic Eq. (10) shows that as $M_0 \to 1$ and for a fixed X the distribution of the longitudinal velocity components u coincides with the sonic. The distribution of the velocity gradient has the form

$$\frac{du}{dX} = -\frac{3}{\delta^2}\frac{\operatorname{sh}\varphi/3}{\operatorname{sh}\varphi}, \quad \operatorname{ch}\varphi = -1 + \frac{27}{2}\frac{X}{\delta^3}, \quad X > \frac{4}{27}\delta^3$$
$$\frac{du}{dX} = -\frac{3}{\delta^2}\frac{\sin(\pi/3 - \varphi/3)}{\sin\varphi}, \quad \cos\varphi = 1 - \frac{27}{2}\frac{X}{\delta^3} \quad 0 \leqslant X \leqslant \frac{4}{27}\delta^3 \tag{11}$$

From (10) and (11) it follows that as $M_0 \to 1$, at distances $X \sim \delta^3$ the ratios of the velocities and of the pressure gradients (the subscript l denotes quantities calculated by the linear theory) are

$$\frac{u}{u_l} \sim 0.75, \quad \left(\frac{du}{dX}\right) \Big/ \left(\frac{du}{dX}\right)_l \sim 0.6 \tag{12}$$

Estimates (12) testify that when $X \sim \delta^3$ the nonlinear term in Eqs. (1) begins to play an essential role. This result is natural since the boundary-value problem (1), (2) admits of a similarity transformation group

$$u = \frac{\varepsilon}{(1+\gamma)M_0^2}\bar{u}, \quad v = \frac{\varepsilon^{3/2}}{(1+\gamma)M_0^2}\bar{v}, \quad x = \varepsilon^3\bar{x}, \quad \psi = \varepsilon^{5/2}\bar{\psi}, \ M_0 \neq 0$$

In the new variables boundary-value problem (1), (2) is independent of the mangitude of number $M_0 \neq 0$ and all terms at distances $x \sim \varepsilon^3$, $\psi \sim \varepsilon^{5/2}$ play a like role. In this connection $u \sim \varepsilon$, $v \sim \varepsilon^{3/2}$, $v \sim u^{3/2}$. The estimates show as well that when $X < (4\delta^3) / 27$ we can use the linear theory without much guilt. The pressure gradient is favorable and as $X \to 0$ can be represented in this interval of variation of x as

$$dp / dx = -(1 + \gamma)^{-1}(-2\varepsilon d_{11}x)^{-1/2} \tag{13}$$

When $X \gtrsim \delta^3$ the essential role in the formation of the pressure gradient begins to be played by the nonlinear term in system (1). When $u > 0$ and $v > 0$ formulas (7) describe the gas flow with a free streamline, separating from the smooth surface. Its form coincides with that of the free streamline in the case of incompressible liquid separation /8/ and is determined by the equation

$$y = {}^2/_3\,(1 + \gamma)^{-1}(2 / d_{11})^{1/2}x^{3/2} + \ldots, \quad x > 0$$

When $u < \varepsilon / M_0^2$ relation (7) can be used in the region $-(d_{11}\varepsilon^3) / (6M_0^4) < x < 0$. However, if $M_0 < (2 + \gamma)^{-1/2}$, then formulas (7) are valid everywhere that the problem's solution can be described by the theory of small perturbations. When $M_0 = 1$ they become meaningless. In this case the medium's compressibility is vital, and in a neighborhood of the separation point there is no region in which the solution could be linearized.

As a result of the interaction of the external potential flow described by formulas (7), (11), (3) with the boundary layer under the action of an infinite pressure gradient in it there is formed a free interaction domain. Its extent Δx depends on the Reynolds number $R = (\rho_0 U_0 L) / \mu_0$ and on M_0. Here L is the characteristic size of the external potential flow, μ is the viscosity coefficient. If $\Delta x \ll (4\delta^3) / 27$, then the flow in the free interaction domain is formed by the pressure gradient (13). Let us determine the dependency of Δx on the numbers R and ε and construct a solution of the Navier—Stokes equations, valid in a complete neighborhood of the corner AOC outside the free interaction domain.

The equation of the free streamline and the pressure gradient (13) depend on variable x

in the same way as in the case of separation of an incompressible liquid's separation from the corner point /2/. Therefore, the dependency of all characteristic sizes of the free interaction domain on R will be analogous /3/. The free interaction domain's structure is three-decked /3,5−8/. By $U(Y)$ and $R(Y)$ we denote the values of the velocity and the density at the corner point O, while by $Y = R^{1/2}y$, the coordinate of the boundary layer. The solution of the system of Navier−Stokes equations in the main deck (region 2 in the Fig.1) is sought as perturbations to the values $U(Y)$ and $R(Y)$:

$$\begin{aligned} q_x &= U(Y) + (-x)^{3/8} u_1(Y) + (-x)^{1/2} u_2(Y) + \ldots \\ V_y &= (-x)^{-5/8} V_1(Y) + (-x)^{-1/2} V_2(Y) + \ldots, \quad V_y = R^{1/2} q_y \\ \rho &= R(Y) + (-x)^{3/8} \rho_1(Y) + (-x)^{1/2} \rho_2(Y) + \ldots \\ p &= 1 + \gamma M_0^2 \left(-\frac{2x}{\varepsilon d_{11}}\right)^{1/2} + \ldots \end{aligned} \tag{14}$$

Expansions (14) describe a nonviscous turbulent flow. The functions u_1, V_1, ρ_1 and u_2, V_2, ρ_2 satisfy ordinary differential equations. Their integration yields

$$\begin{aligned} &u_1 = \frac{8}{3} A_1 U'(Y), \qquad \rho_1 = \frac{8}{3} A_1 R'(Y), \qquad V_1 = A_1 U(Y) \\ &u_2 = 2\left[A_2 + \frac{1}{2} d_0 J(Y)\right] U'(Y) - \frac{d_0}{R(Y)\,U(Y)}, \qquad d_0 = (2/\varepsilon d_{11})^{1/4} \\ &\rho_2 = 2\left[A_2 + \frac{1}{2} d_0 J(Y)\right] \cdot R'(Y) + d_0 R(Y) \\ &V_2 = \left[A_2 + \frac{1}{2} d_0 J(Y)\right] U(Y), \qquad J(Y) = \int_Y^\infty \frac{1 - RU^2}{RU^2}\, dY - \varepsilon Y \end{aligned} \tag{15}$$

However, expansions (14) with solutions (15) do not ensure the fulfillment of the adhesion boundary condition. Therefore, it is necessary to consider a viscous sublayer (domain 1) in which the forces of viscosity, inertia and pressure are of a like order of smallness. In it we seek the solutions in the form

$$\begin{aligned} q_x &= (-x)^{1/4} u_0(\eta) + (-x)^{1/2} u_1(\eta) + \ldots, \\ p &= 1 + \gamma M_0^2 d_0 (-x)^{1/2} + \ldots, \\ \rho &= R(0) + (-x)^{1/4} \rho_0(\eta) + (-x)^{1/2} \rho_1(\eta) + \ldots, \\ V_y &= (-x)^{-3/8} V_0(\eta) + (-x)^{1/8} V_1(\eta) + \ldots \end{aligned} \tag{16}$$

We assume that the viscosity coefficient is proportional to the temperature: $\mu = CT$. Using (16) and the state equation, we obtain

$$T = \frac{1}{R(0)}\left[1 - (-x)^{1/4} \frac{\rho_0(\eta)}{R(0)} + (-x)^{1/2}\left(\frac{\rho_0^2(\eta)}{R^2(0)} - \frac{\rho_1(\eta)}{R(0)} + \gamma M_0^2 d_0\right) + \ldots\right]$$

If we introduce the stream function

$$\begin{aligned} &\Psi = [(-x)^{5/8} F_0(\eta) + (-x)^{7/8} F_1(\eta) + \ldots]\, R(0), \qquad \eta = \frac{Y}{(-x)^{3/8}} \\ &u_0 = \frac{dF_0}{d\eta}, \qquad V_0 = \frac{5}{8} F_0 - \frac{3}{8} \eta \frac{dF_0}{d\eta} \\ &u_1 = \frac{dF_1}{d\eta} - \frac{\rho_0 u_0}{R(0)}, \qquad V_1 = \frac{7}{8} F_1 - \frac{3}{8} \eta \frac{dF_0}{d\eta} - \frac{\rho_0 V_0}{R(0)} \end{aligned}$$

then the equations for determining the flow field in the viscous sublayer are

$$\begin{aligned} &\frac{C}{R^2(0)} \frac{d^3F_0}{d\eta^3} - \frac{5}{8} F_0 \frac{d^2F_0}{d\eta^2} + \frac{1}{4}\left(\frac{dF_0}{d\eta}\right)^2 = -\frac{1}{2} \frac{d_0}{R(0)} \\ &\frac{C}{PrR^2(0)} \frac{d^2\rho_0}{d\eta^2} - \frac{5}{8} F_0 \frac{d\rho_0}{d\eta} + \frac{1}{4} \frac{dF_0}{d\eta} \rho_0 = 0 \\ &\frac{C}{R^2(0)} \frac{d^3F_1}{d\eta^3} - \frac{5}{8} F_0 \frac{d^2F_1}{d\eta^2} + \frac{3}{4} \frac{dF_0}{d\eta} \frac{dF_1}{d\eta} - \frac{7}{8} \frac{d^2F_0}{d\eta^2} F_1 = \\ &\frac{1}{2} \frac{d_0}{R(0)} \rho_0 + \frac{C}{R^2(0)} \rho_0 \frac{d^3F_0}{d\eta^3} - \frac{C}{PrR^2(0)} \frac{dF_0}{d\eta} \frac{d^2\rho_0}{d\eta^2} + \\ &\frac{C}{R^3(0)}\left[\frac{d}{d\eta}\left(\rho_0 \frac{d^2F_0}{d\eta^2}\right) + \frac{d^2}{d\eta^2}\left(\rho_0 \frac{dF_0}{d\eta}\right)\right] \\ &\frac{C}{PrR^2(0)} \frac{d^2\rho_1}{d\eta^2} - \frac{5}{8} F_0 \frac{d\rho_1}{d\eta} + \frac{1}{2} \frac{dF_0}{d\eta} \rho_1 = \frac{1}{2} M_0^2 d_0 R(0) \frac{dF_0}{d\eta} - \end{aligned} \tag{17}$$

$$\frac{1}{4}\rho_0\frac{dF_1}{d\eta}+\frac{7}{8}F_1\frac{d\rho_0}{d\eta}+\frac{C}{PrR^3(0)}\rho_0\frac{d^2\rho_0}{d\eta^2}+$$
$$C(\gamma-1)M_0^2\left(\frac{dF_0}{d\eta}\right)^2+\frac{3C}{PrR^3(0)}\left(\frac{d\rho_0}{d\eta}\right)^2$$

The boundary conditions for them are the following. As $\eta\to\infty$ expansions (14) and (16) must be combined. The adhesion conditions requires that $F_0=F_0'=F_1=F_1'=0$ when $\eta=0$. If the temperature of the corner's surface AO is constant, then $\rho_0(0)=0,\ \rho_1(0)=\gamma M_0^2 d_0 R(0)$. We note that $d\rho_0/d\eta\neq 0$ when $\eta=0$. If, however, the corner's surface is heat-insulated, then it is necessary to require $\rho_1'(0)=0,\ \rho_0(\eta)\equiv 0$. From the latter condition as well it follows that $F_1(\eta)\equiv 0$.

Let us consider the case when the corner's surface is heat-insulated and the Prandtl number $\mathrm{Pr}=1$. As $\eta\to\infty$ the solutions of Eqs.(17), behaving algebraically, can be represented as (b_0, b_1, c_0, c_1 are arbitrary constants)

$$F_0=b_0\eta^{5/3}+b_1\eta^{2/3}+\ldots,\quad \rho_1=c_0\eta^{4/3}+\ldots$$

Thus, the solutions of Eqs.(17) for F_0 and ρ_1 must satisfy the conditions

$$F_0=F_0'=0,\quad \rho_1'(\eta)=0,\quad \eta=0$$
$$F_0=O(\eta^{5/3}),\quad \rho_1=O(\eta^{4/3}),\quad \eta\to\infty \tag{18}$$

It was shown in /9/ that the boundary-value problem (17), (18) for F_0 has a unique solution. It can be verified as well that in case $\mathrm{Pr}=1$ the integral

$$\rho_1=\gamma d_0 R(0) M_0^2+\frac{\gamma-1}{2}R^2(0)M_0^2\left(\frac{dF_0}{d\eta}\right)^2 \tag{19}$$

satisfies Eq.(17) for ρ_1 and the boundary conditions (18). Let us determine the dependent of coefficients b_0 and b_1 on ε. To do this we make an expansion transformation $\eta=\alpha\zeta$, $F_0=\beta\Phi_0$ with coefficients

$$\alpha=2^{1/3}R(0)^{-3/4}d_{11}^{1/3}\varepsilon^{1/3}C^{1/2},\ \beta=2^{11/3}R(0)^{-3/4}d_{11}^{-1/3}\varepsilon^{-1/3}C^{1/2}$$

In the new variables the first equation in (17) and the boundary conditions for it in (18) take the form

$$\frac{5}{2}\Phi_0\frac{d^2\Phi_0}{d\zeta^2}-\left(\frac{d\Phi_0}{d\zeta}\right)^2-\frac{d^3\Phi_0}{d\zeta^3}=1$$
$$\Phi_0(0)=\Phi'(0)=0,\quad \Phi_0(\zeta)=B_0\zeta^{5/3}+B_1\zeta^{2/3}+\ldots,$$
$$\zeta\to\infty$$

The constants B_0 and B_1 are already independent of ε. Going back to the former variables, we obtain, as $\eta\to\infty$, the following expansion of the stream function

$$\Psi=2^{1/3}d_{11}^{-1/3}C^{-1/3}B_0\varepsilon^{-1/3}Y^{5/3}+2^{17/24}R(0)^{-4/3}d_{11}^{-5/24}C^{1/3}\varepsilon^{-5/24}B_1(-x)^{5/8}Y^{2/3}+\ldots \tag{20}$$

Combining expansions (19) and (20) with expansions (14) yields

$$U(Y)=\frac{2^{1/3}5}{3d_{11}^{1/3}}C^{-1/3}\varepsilon^{-1/3}B_0Y^{2/3}+\ldots,\quad A_1=\frac{9d_{11}^{1/3}C^{1/2}}{5\,2^{41/24}R(0)^{4/3}}\varepsilon^{1/3}\frac{B_1}{B_0}$$
$$R(Y)=R(0)+(\gamma-1)R^2(0)M_0^2\,\frac{2^{-1/3}5^2C^{-2/3}}{3^2d_{11}^{2/3}}\varepsilon^{-2/3}B_0^2Y^{4/3}+\ldots,\ Y\to 0 \tag{21}$$

As $Y\to\infty$ the functions $U(Y)\to 1$, $R(Y)\to 1$; therefore, the boundary layer induces perturbations of order $R^{-1/2}$ in the external stream (domain 3):

$$q_y=R^{-1/2}[A_1(-x)^{-3/8}+A_2(-x)^{-1/2}+\ldots],\quad Y\to\infty \tag{22}$$

The flow in the external stream is described by formulas (9). Thanks to the displacing influence of the boundary layer, in expansion (9) it is necessary to introduce a new term with $\omega=-{}^1/_4$. The constant equals

$$D_{-1/4}^{(a)}=-2^{5/4}d_{11}^{3/8}A_1\varepsilon^{-1/2}R^{-1/2}$$

The connection between constants $D_{-1/4}^{(a)}$ and $D_{-1/4}^{(s)}$ is determined from the condition of equality to zero of the free streamline:

$$D^{(s)}_{-1/4} = -\operatorname{ctg}\pi/8\, D^{(a)}_{-1/4}$$

The constant A_2 in (15) and (22) needs to be set equal to zero since the solution connected with it is, in the external stream, a natural solution of the corner flow-around problem with a free streamline.

The boundary layer which is separating forms a mixing layer (domains 4 and 5) which abuts, on the one side, the stagnation zone (domain 6) and, on the other, the external potential flow. Since the stream is nonviscous in the main part of the boundary layer, the changes in velocity, density and pressure along each streamline are infinitesimal as $R \to \infty$. This signifies that in the main part of the mixing layer (domain 4) the solution needs to be sought, as in the boundary layer, in the form of perturbations to values $U(Y)$ and $R(Y)$ whose behavior as $Y \to 0$ is given by formulas (21). Starting from this, we seek the solution in the viscous sublayer (domain 5) as

$$\Psi = R(0)\,[x^{\circ 5/8} F_0(\eta) + \ldots], \quad \eta = Y^\circ / x^{\circ 3/8} \tag{23}$$
$$\rho = R(0) + x^{\circ 1/4} \rho_1(\eta) + \ldots$$

By x°, Y° we mean an orthogonal coordinate system whose x° -axis coincides with the free streamline. For the determination of F_0 and ρ_1 we obtain the equations

$$\frac{C}{R^2(0)} \frac{d^3F_0}{d\eta^3} + \frac{5}{8} F_0 \frac{d^2F_0}{d\eta^2} - \frac{1}{4}\left(\frac{dF_0}{d\eta}\right)^2 = 0 \tag{24}$$
$$\frac{C}{R^2(0)} \frac{d^2\rho_1}{d\eta^2} + \frac{5}{8} F_0 \frac{d\rho_1}{d\eta} - \frac{1}{2} \frac{dF_0}{d\eta}\rho_1 = C(\gamma - 1) M_0^2 \left(\frac{d^2F_0}{d\eta^2}\right)^2$$

As $\eta \to \infty$ expansions (23) must be combined with expansions (20) and (21), while as $\eta \to -\infty$, with the expansions describing the flow in the stagnation zone. For function F_0 this gives

$$F_0 = b_0 \eta^{5/3} + \frac{2C}{45R^2(0)} \eta^{-1} + \ldots, \quad \eta \to \infty \tag{25}$$
$$F_0(0) = 0, \quad F_0'(\eta) \to 0, \quad \eta \to -\infty$$

Boundary-value problem (24), (25) was solved numerically in /3/. The equation for the density ρ_1 in case $\Pr = 1$ can be integrated directly

$$\rho_1 = \frac{\gamma - 1}{2} R^2(0) M_0^2 \left(\frac{dF_0}{d\eta}\right)^2$$

For the vertical velocity component we have as $\eta \to -\infty$

$$q_y = -{}^5/_8 F_0(-\infty)\, x^{-3/8} R^{-1/2} + \ldots$$

Since it does not equal zero /3/, the motion

$$q_x = {}^5/_8 F_0(-\infty) \frac{\cos({}^5/_8\alpha - {}^3/_8\varphi)}{\sin {}^5/_8\alpha} r^{-3/8} R^{-1/2} \tag{26}$$
$$q_y = -{}^5/_8 F_0(-\infty) \frac{\sin({}^5/_8\alpha - {}^3/_8\varphi)}{\sin {}^5/_8\alpha} r^{-3/8} R^{-1/2}, \quad -\alpha \leqslant \varphi \leqslant 0$$

is induced in the stagnation zone. The pressure and the density are found from the relations

$$p = \left[1 - \frac{25}{128}(\gamma - 1) M_0^2 R(0) F_0^2(-\infty) \frac{r^{-3/4}}{\sin^2 {}^5/_8\alpha} R^{-1}\right]^{\gamma/(\gamma-1)}$$
$$\rho = p^{1/\gamma} R(0)$$

Solution (26) does not satisfy the adhesion condition on the corner's surface OC. Along it the velocity equals

$$-{}^5/_8 F_0(-\infty)\, r^{-3/8} \sin^{-1} {}^5/_8\alpha\, R^{-1/2}$$

The boundary layer on the corner's surface OC (domain 7) is constructed in the usual manner. Thus, the corner AOC has been completely bypassed and the expansions of the solution, constructed in each domain, have been intercombined. However, when $x \sim \Delta x$ they cease to be valid.

Let us consider the pressure's expansion when $x < 0$:

$$p = 1 + \gamma M_0^2 [(2/\varepsilon d_{11})^{1/2} (-x)^{1/2} + 2^{-5/8} d_{11}^{-3/8} D^{(s)}_{-1/4} (-x)^{-1/8} + \ldots]$$

The distance at which the second term becomes of the same order as the first gives the typical size Δx of the free interaction domain. The linear theory of the external potential flow is true if $\Delta x \leqslant (d_{11}\varepsilon^3)/6$. Hence we get that the boundary layer will separate, as in an incompressible liquid, if

$$\varepsilon^3 \gg \frac{3^{75/26}\operatorname{ctg}^{12/13}\pi/8C^{6/13}}{5^{12/26}2d_{11}^{6/13}R(0)^{16/13}}\left(\frac{B_1}{B_0}\right)^{12/13} R^{-6/13} \tag{27}$$

As $\varepsilon \to 0$ the Reynolds number will tend to a value computed from the critical mangnitudes of the parameters. The free interaction domain will be formed in conformity with the pressure gradient given by the first formula in (11), while its extent will approximate to its own sonic value $\Delta x_* = O(R^{-3/7})$. If, however, the displacement's thickness is formed by a viscous sublayer, then $\Delta x_* = O(R^{-6/13})$ /10/ and the order of Δx_* coincides with the order of the right-hand side of inequality (27).

REFERENCES

1. LIEPMANN H.W. and ROSHKO A., Elements of Gasdynamics. New York, J. Wiley and Sons., Inc., 1957.
2. ACKERBERG R.C., Boundary-layer separation at a free stream line. Pt 1. Two-dimensional flow. J. Fluid Mech., Vol.44, pt 2, 1970.
3. RUBAN A.I., On the theory of laminar separation of a liquid from a fracture point of a solid surface. Uch. Zap. Tsentral'n. Aero-Gidrodinamichesk. Inst., Vol.7, No.4, 1976.
4. DIESPEROV V.N., On a solution of Kármán's equation, describing flow past a convex corner. Dokl. Akad. Nauk SSSR, Vol.254, No.6, 1980.
5. NEILAND V. Ia., On the theory of separation of a laminar boundary layer in a supersonic stream. Izv. Akad. Nauk SSSR, Mekhan. Zhidk. Gaza, No.4, 1969.
6. STEWARTSON K. and WILLIAMS P.G., Self-induced separation. Proc. Roy. Soc. Lond. Ser. A., Vol.312, No. 1509, 1969.
7. MESSITER A.F., Boundary-layer flow near the trailing edge of a flat plate. SIAM J. Appl. Math., Vol.18, No.1, 1970.
8. SYCHEV V.V., On laminar separation. Izv. Akad. Nauk SSSR, Mekhan. Zhidk. Gaza, No.3, 1972.
9. McLEOD J.B., The existence and uniqueness of a similarity solution arising from separation at a free stream line. Quart. J. Math., Vol.23, No.89, 1972.
10. DIESPEROV V.N., On the structure of the boundary layer under a transonic flow around a convex corner with a free streamline. Dokl. Akad. Nauk SSSR, No.6, 1981.

Translated by N.H.C.